SAFE HANDS & IIT-ian's PACE

LEAP TEST# 03 (JEE) ANS KEY Dt. 05-12-2023

PHYSICS		CHEMISTRY		MATHS	
Q. NO.	[ANS]	Q. NO.	[ANS]	Q. NO.	[ANS]
1	A	31	B	61	D
2	A	32	B	62	D
3	A	33	A	63	C
4	A	34	C	64	A
5	C	35	C	65	B
6	A	36	D	66	A
7	C	37	B	67	C
8	D	38	A	68	B
9	C	39	D	69	D
10	B	40	A	70	A
11	C	41	B	71	D
12	B	42	C	72	D
13	B	43	C	73	B
14	D	44	B	74	C
15	C	45	A	75	B
16	C	46	C	76	B
17	B	47	A	77	A
18	D	48	B	78	D
19	D	49	A	79	D
20	C	50	A	80	C
21	6	51	6	81	1
22	4	52	18	82	1
23	5	53	3	83	0
24	2	54	2	84	2
25	2	55	2.83	85	4

See Physics and Mathematics solutions on next page....

SAFE HANDS & PACE

Leap Test 03 (JEE) Physics Solutions

: ANSWER KEY :

1)	a	2)	a	3)	a	4)	a	17)	b	18)	d	19)	d	20)	c
5)	c	6)	a	7)	c	8)	d	21)	6	22)	4	23)	5	24)	2
9)	c	10)	b	11)	c	12)	b	25)	2						
13)	b	14)	d	15)	c	16)	c								

: HINTS AND SOLUTIONS :

Single Correct Answer Type

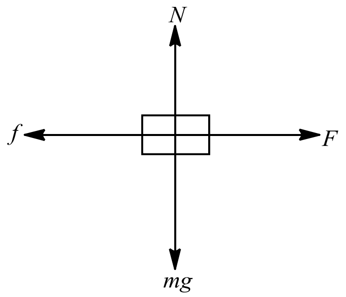
1 **(a)**

The mass m is not moving with respect to the lift and also has no tendency to move. Hence, friction force acting on it is equal to zero

2 **(a)**

The various forces acting on the block are as shown

As the truck moves in forward direction with acceleration $2\ \mathrm{m/s^2}$, the box experiences a force F



in backward direction,

$F = ma = 40 \times 2 = 80$ N

in backward direction.

Its motion will be opposed by force of friction

$f = \mu N = \mu mg = 0.15 \times 40 \times 10 = 60$ N

The acceleration of the box relative to the truck toward the rear end is

$$a = \frac{F - f}{m} = \frac{80 - 60}{40} = 0.5\ m/s^2$$

If t be the time taken by the box to fall off the truck

$$s = ut + \frac{1}{2}at^2$$

$$5 = 0 + \frac{1}{2} \times 0.5 \times t^2$$

$$t = \sqrt{20}\ s$$

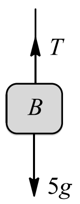
During this time distance covered by truck

$$x = 0 \times t + \frac{1}{2} \times 2 \times (\sqrt{20})^2 = 20\ \mathrm{m}$$

3 **(a)**

Since the given system is in equilibrium therefore tension in the string is 5g

When we consider the combination of A and C , then



$T = \mu R$

or $T = 0.2(10 + \text{mass of C})\mathrm{g}$

or $5\mathrm{g} = 0.2(10 + \text{mass of C})\mathrm{g}$

or mass of $C = 15$ kg

4 **(a)**

When the string C is stretched slowly, the tension in A is greater than that of C, because of the weight mg and the former reaches breaking point earlier

5 **(c)**

As the spring balances are massless therefore the reading of both balance should be equal

7 **(c)**

Let coefficient of friction is μ, and then retardation will be $\mu\, g$.

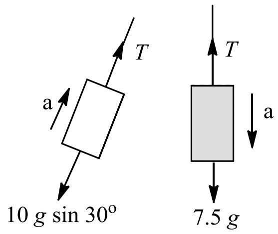
From equation of motion, $v = u + at$

$$\Rightarrow \quad 0 = 6 - \mu\, g \times 10$$

$$\Rightarrow \quad \mu = \frac{6}{100} = 0.06$$

8 **(d)**

Refer to the free-body diagrams



$T - 10\mathrm{g}\sin 30° = 10\alpha$ or $T - 5\mathrm{g} = 10\alpha$

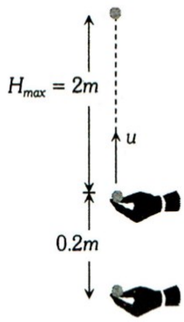
Again , $7.5 - T = 7.5\alpha$

Adding, $2.5\mathrm{g} = 17.5\alpha$

or $\alpha = \frac{25\mathrm{g}}{175} = \frac{\mathrm{g}}{7}$

9 **(c)**

Let the ball starts moving with velocity 'u' and it reaches up to maximum height $H_{\max}$, then



From $H_{\max} = \frac{u^2}{2g}$

$$u = \sqrt{2g\,(H_{\max})}$$

$$= \sqrt{2 \times 10 \times 2} = 2\sqrt{10}\ m/s$$

This velocity is supplied to the ball by the hand and initially the hand was at rest,

it acquires this velocity in distance of $0.2\ meter$

$\therefore\ a = \dfrac{u^2}{2s} = \dfrac{40}{2 \times 0.2} = 100\ m/s^2$

So upward force on the ball $F = m(g + a)$

$= 0.2(10 + 100) = 0.2 \times 110 = 22N$

10 **(b)**

When a sudden jerk is given to C, an impulsive tension exceeding the breaking tension develops in C first, which breaks before this impulse can reach A as a wave through block

11 **(c)**

From Newton's second law of motion,

$\text{Force} = \text{mass} \times \text{acceleration}$

Acceleration along y-axis is

$a_y = -\dfrac{5}{5}\text{ms}^{-2} = -1\text{ms}^{-2}$

Form equation of motion

$v_y = u_y + a_y t$

Where v_y is initial velocity along y-axis, t the time and v_y the final velocity along y-axis.

Given, $v_y = 0, u_y = 40\ \text{ms}^{-1}, \quad a_y = -1\ \text{ms}^{-1}$

$\therefore\ 0 = 40 - 1 \times t$

$\Rightarrow t = 40\ \text{s}$

12 **(b)**

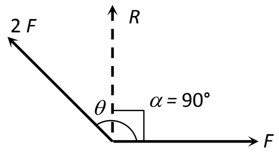
$F = \dfrac{d}{dt}(p)$

$F = \frac{d}{dt}(a + bt + ct^2)$ or $F = b + 2ct$

Clearly, the force is time-dependent

13 **(b)**

$\tan\alpha = \frac{2F\sin\theta}{F+2F\cos\theta} = \infty$ (as $\alpha = 90°$)



$\Rightarrow F + 2F\cos\theta = 0$

$\Rightarrow \cos\theta = -\dfrac{1}{2}$

$\theta = 120°$

14 **(d)**

Given that, $\dfrac{dm}{dt} = 0.1\ \text{kgs}^{-1}$;

mass of the rocket=100 kg

and $v = 1\ \text{kms}^{-1} = 1000\ \text{ms}^{-1}$

Thrust on the rocket, $F = v\dfrac{dm}{dt} = 1000 \times 0.1$

Now, $F = Ma$

$\therefore\ a = \dfrac{1000 \times 0.1}{100} = 1\ \text{ms}^{-2}$

15 **(c)**

Mass measured by physical balance remains unaffected due to variation in acceleration due to gravity

16 **(c)**

Retardation = $g(\sin 60° + \mu\cos 60°)$

$= 10\left(\dfrac{\sqrt{3}}{2} + \mu\dfrac{1}{2}\right) = 5(\sqrt{3} + \mu)$

$v = u - at$

$0 = 20 - 5(\sqrt{3} + \mu) \times 2$

$\mu = 2 - 1.732 \approx 0.27$

17 **(b)**

From the graph, it is a straight line so, uniform motion. Because of impulse direction of velocity changes as can be seen from the slope of the graph.

Initial velocity $\mathbf{v}_1 = \dfrac{2}{2} = 1\ \text{ms}^{-1}$

Final velocity $\mathbf{v}_2 = \dfrac{-2}{2} = -1\ \text{ms}^{-1}$

$\mathbf{p}_i = m\mathbf{v}_1 = 0.4\ \text{Ns}$

$\mathbf{p}_f = m\mathbf{v}_2 = -0.4\ \text{Ns}$

$\mathbf{J} = \mathbf{p}_f - \mathbf{p}_i = -0.4 - 0.4$

$= -0.8\ Ns\ \ (\boldsymbol{J} = impulse)$

$\therefore\ |\mathbf{J}| = 0.8\ \text{Ns}$

18 **(d)**

$T = \dfrac{2m_1m_2}{m_1 + m_2}g = \dfrac{2 \times 10 \times 6}{10 + 6} \times 9.8 = 73.5\ N$

19 **(d)**

$v = u - at \Rightarrow u - \mu gt = 0\ \therefore\ \mu = \dfrac{u}{gt} = \dfrac{6}{10 \times 10}$

$= 0.06$

20 **(c)**

For given condition $s \propto \frac{1}{m^2}$

$\therefore \dfrac{s_2}{s_1} = \left(\dfrac{m_1}{m_2}\right)^2 = \left(\dfrac{200}{300}\right)^2$

$\Rightarrow s_2 = s_1 \times \dfrac{4}{9} = 36 \times \dfrac{4}{9} = 16\ m$

Integer Answer Type

21 **(6)**

Force of friction between the two will be maximum i.e.,

μmg. Retardation of A is $a_A = \frac{\mu mg}{m} = \mu g$

And acceleration of B is $a_B = \frac{\mu mg}{2m} = \frac{\mu g}{2}$

Acceleration of B relative to A is $a_{BA} = a_A + a_B = \frac{3\mu g}{2}$

Substituting, $\mu = \frac{1}{2}$; $a_{BA} = \frac{3g}{4}$

22 **(4)**

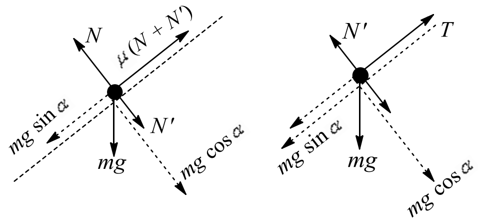
Since A tends to slip down, frictional forces act on it from both sides up the plane

Let N be the reaction of the plank on A and N' be the mutual normal action-reaction between A and B

From the free-body diagram of A

$N' + mg\cos\alpha = N$ and $mg\sin\alpha = \mu(N + N')$

From the free-body diagram of B



$N6'' = mg\cos\alpha$

$mg\sin\alpha + \mu N' = T$

$\therefore\ 2\,mg\cos\alpha = N$

and $mg\sin\alpha = \mu(2\,mg\cos\alpha + mg\cos\alpha)$

or $\mu = \frac{1}{3}\tan\alpha = \frac{1}{3}\times\frac{3}{4} = 0.25$ or 1/m=4

23 **(5)**

Denote the common magnitude of the maximum acceleration as$a$. For block A to remain at rest with respect to block A to remain at rest with respect to block $B, a \le \mu_s g$. to be largest. The tension in the cord is then

$T = (m_A + m_B)a + \mu_k g(m_A + m_B)$

$= (m_A + m_B)(a + \mu_k g)$

This tension is related to the mass m_C (largest) by $T = m_C(g - a)$.Solving for m_C yields

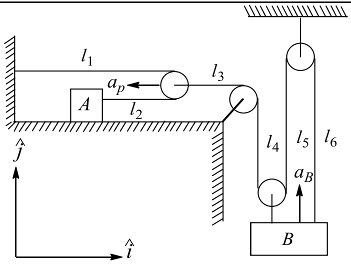
$$m_C = \frac{(m_A + m_B)(\mu_s + \mu_k)}{1 - \mu_s} = \frac{(1.5 + 0.5)(0.6 + 0.4)}{1 - 0.6}$$

$= 5\text{ kg}$

24 **(2)**

$\ell_1 + \ell_2 = C \Rightarrow \ell_1' + \ell_2' = 0$

$\Rightarrow -a_p + (12 - a_p) = 0 \Rightarrow a_p = 6\,ms^{-2}$



$\ell_3 + \ell_4 + \ell_5 + \ell_6 = C \Rightarrow \ell_3' + \ell_4' + \ell_5' + \ell_6' = 0$

$a_P - a_B - a_B - a_B = 0 \Rightarrow a_P = 3a_B \Rightarrow a_B = 2\text{ ms}^{-1}$

25 **(2)**

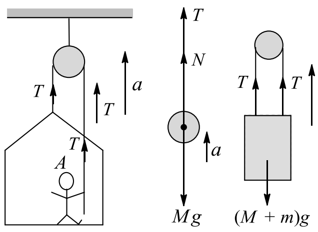
Let M = mass of painter =10 kg

m = mass of crate=25 kg

Let F_A be the action force exerted by painter on crate, reaction force exerted by crate on man

$N = F_A = 450$ N

The free-body diagram of painter is shown in figure (b)



(a) (b) (c)

Therefore, equation of motion of painter is

$N + T - Mg = Ma$ (i)

The equation of motion of whole system is

$2T - (M + m)g = (M + m)a$ (ii)

Multiplying (i) by 2, we get

$2N + 2T - 2Mg = 2Ma$ (iii)

Subtracting (ii) from (iii), we get

$2N - 2Mg + (M + m)g = (2M - M - m)a$

or $2N - (M - m)g = (M - m)a$

$$a = \frac{2N - (M - m)g}{M - m}$$

$$= \frac{2 \times 450 - (100 - 25) \times 10}{100 - 25} = \frac{900 - 750}{75} = 2\text{ ms}^{-2}$$

: ANSWER KEY :

61)	d	62)	d	63)	c	64)	a	77)	a	78)	d	79)	d	80)	c
65)	b	66)	a	67)	c	68)	b	81)	1	82)	1	83)	0	84)	2
69)	d	70)	a	71)	d	72)	d	85)	4						
73)	b	74)	c	75)	b	76)	b								

: HINTS AND SOLUTIONS :

Single Correct Answer Type

61 **(d)**

$A(\alpha)=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

$A(\beta)=\begin{bmatrix}\cos\beta & \sin\beta\\ -\sin\beta & \cos\beta\end{bmatrix}$

$A(\alpha).A(\beta)=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\beta & \sin\beta\\ -\sin\beta & \cos\beta\end{bmatrix}$

A(α).A(β)=

$\begin{bmatrix}\cos\alpha\sin\beta-\sin\alpha\sin\beta & \cos\alpha\sin\beta+\sin\alpha\cos\beta\\ -\sin\alpha\cos\beta-\cos\alpha\sin\beta & -\sin\alpha\ \sin\beta\ +\ \cos\alpha\cos\beta\end{bmatrix}$

$A(\alpha).A(\beta)=\begin{bmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta)\\ -\sin(\alpha+\beta) & \cos(\alpha+\beta)\end{bmatrix}$

$A(\alpha).A(\beta)=A(\alpha+\beta)$

62 **(d)**

$(AB^{-1}C)^{-1}=C^{-1}(B^{-1})^{-1}A^{-1}=C^{-1}BA^{-1}$

63 **(c)**

We have, $\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}A=\begin{bmatrix}1 & 1\\ 0 & -1\end{bmatrix}$

$\Rightarrow A=\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}^{-1}\begin{bmatrix}1 & 1\\ 0 & -1\end{bmatrix}$

$\Rightarrow\quad A=\begin{bmatrix}1 & -3\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & -1\end{bmatrix}$

$=\begin{bmatrix}1 & 4\\ 0 & -1\end{bmatrix}$

64 **(a)**

We have,

$A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}$

$\Rightarrow \frac{1}{n}A^n=\begin{bmatrix}\frac{\cos n\theta}{n} & \frac{\sin n\theta}{n}\\ \frac{-\sin n\theta}{n} & \frac{\cos n\theta}{n}\end{bmatrix}$

$\Rightarrow \lim_{n\to\infty}\frac{1}{n}A^n=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

65 **(b)**

$\because$ satisfied by $x=1, y=-1, z=1$

66 **(a)**

Given system of equations is $x+2y+3z=1,\ 2x+y+3z=2$ and $5x+5y+9z=5$

Now, $\Delta=\begin{vmatrix}1 & 2 & 3\\ 2 & 1 & 3\\ 5 & 5 & 9\end{vmatrix}$

$=1(9-15)-2(18-15)+3(10-5)$

$=-6-6+15$

$=3\neq 0$

Hence, it has unique solution

67 **(c)**

Given, $A=\begin{bmatrix}3 & 4\\ 5 & 7\end{bmatrix}$

$\Rightarrow |A|=1$

$\therefore A\ \text{adj}\ (A)=\begin{bmatrix}3 & 4\\ 5 & 7\end{bmatrix}\begin{bmatrix}7 & -4\\ -5 & 3\end{bmatrix}$

$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=1\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=|A|I$

68 **(b)**

Here $x=1, y^2=4$

$\therefore x=1, y=\mp 2$

$\therefore (x,y)=(1,2)\ \&\ (x,y)=(1,-2)$

69 **(d)**

$A=\begin{bmatrix}x & -2\\ 3 & 7\end{bmatrix}$

$\text{adj A}=\begin{bmatrix}7 & 2\\ -3 & x\end{bmatrix}$

$|A|=7x+6$

$A^{-1}=\frac{\text{adj A}}{|A|}=\begin{bmatrix}\frac{7}{(7x+6)} & \frac{2}{(7x+6)}\\ \frac{-3}{7x+6} & \frac{x}{7x+6}\end{bmatrix}\quad \ldots(i)$

$A^{-1}=\begin{bmatrix}\frac{7}{34} & \frac{1}{17}\\ \frac{-3}{34} & \frac{2}{17}\end{bmatrix}\quad \ldots.(ii)[\text{Given})$

From (i) and (ii), we get

$\begin{bmatrix}\frac{7}{7x+6} & \frac{2}{7x+6}\\ \frac{-3}{7x+6} & \frac{x}{7x+6}\end{bmatrix}=\begin{bmatrix}\frac{7}{34} & \frac{1}{17}\\ \frac{-3}{34} & \frac{2}{17}\end{bmatrix}$

$\Rightarrow \frac{7}{7x+6}=\frac{7}{34}\Rightarrow 7x+6=34\Rightarrow x=4$

70 **(a)**

Let $A=\begin{bmatrix}0 & 1 & -1\\ -1 & 0 & 2\\ 1 & -2 & 0\end{bmatrix}$

$\therefore\quad |A|=\begin{vmatrix}0 & 1 & -1\\ -1 & 0 & 2\\ 1 & -2 & 0\end{vmatrix}$

$=0\begin{vmatrix}0 & 2\\ -2 & 0\end{vmatrix}-1\begin{vmatrix}-1 & 2\\ 1 & 0\end{vmatrix}-1\begin{vmatrix}-1 & 0\\ 1 & -2\end{vmatrix}$

$=0+2-2=0$

$\Rightarrow\ |A|=0$

Now, $(\text{adj}\ A)B=\begin{bmatrix}4 & 2 & 2\\ 2 & 1 & 1\\ 2 & 1 & -1\end{bmatrix}\begin{bmatrix}1\\ -2\\ 3\end{bmatrix}$

$=\begin{bmatrix}4-4+6\\ 2-2+3\\ 2-2-3\end{bmatrix}=\begin{bmatrix}6\\ 3\\ -3\end{bmatrix}\neq O$

$\therefore$ This system of equation is inconsistent, so it has no solution

71 **(d)**

$\begin{bmatrix}7 & -6 & 13\\ 2 & 3 & 4\end{bmatrix}\begin{bmatrix}2\\ -1\\ 1\end{bmatrix}=\begin{bmatrix}x\\ y\end{bmatrix}$

$\therefore \begin{bmatrix}33\\ 5\end{bmatrix}=\begin{bmatrix}x\\ y\end{bmatrix}$

72 **(d)**

$AA^{-1} = I$

$$\begin{bmatrix}0&1&2\\1&2&3\\3&1&1\end{bmatrix}A^{-1} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix}1&2&3\\0&1&2\\3&1&1\end{bmatrix}A^{-1} = \begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}$$

$R_3 - 3R_1$

$$\begin{bmatrix}1&2&3\\0&1&2\\0&-5&-8\end{bmatrix}A^{-1} = \begin{bmatrix}0&1&0\\1&0&0\\0&-3&1\end{bmatrix}$$

$R_1 - 2R_2$

$$\begin{bmatrix}1&0&-1\\0&1&2\\0&-5&-8\end{bmatrix}A^{-1} = \begin{bmatrix}-2&1&0\\1&0&0\\0&-3&1\end{bmatrix}$$

$R_3 + 5R_2$

$$\begin{bmatrix}1&0&-1\\0&1&2\\0&0&2\end{bmatrix}A^{-1} = \begin{bmatrix}-2&1&0\\1&0&0\\5&-3&1\end{bmatrix}$$

$R_3\left(\frac{1}{2}\right)$

$$\begin{bmatrix}1&0&-1\\0&1&2\\0&0&1\end{bmatrix}A^{-1} = \begin{bmatrix}-2&1&0\\1&0&0\\5/2&-3/2&1/2\end{bmatrix}$$

$R_1 + R_3, R_2 - 2R_3$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}A^{-1} = \begin{bmatrix}\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\-4&3&-1\\\frac{5}{2}&-\frac{3}{2}&\frac{1}{2}\end{bmatrix}$$

$$A^{-1} = \begin{bmatrix}\frac{1}{2}&\frac{-1}{2}&\frac{1}{2}\\-4&3&-1\\\frac{5}{2}&-\frac{3}{2}&\frac{1}{2}\end{bmatrix}$$

73 **(b)**

$$A = \begin{vmatrix}-1&2&5\\2&-4&a-4\\1&-2&a+1\end{vmatrix} = \begin{vmatrix}0&0&a+6\\0&0&-a-6\\1&-2&a+1\end{vmatrix}$$

[using $R_1 \to R_1 + R_3$ and $R_2 \to R_2 - 2R_3$]

$$= \begin{vmatrix}0&0&0\\0&0&-a-6\\1&-2&a+1\end{vmatrix} \quad [\text{using} R_1 \to R_1 + R_2]$$

When $a = -6$, $A = \begin{vmatrix}0&0&0\\0&0&0\\1&-2&-5\end{vmatrix}$ $\therefore \rho(A) = 1$

When $a = 6$, $A = \begin{vmatrix}0&0&0\\0&0&-12\\1&-2&7\end{vmatrix}$, $\therefore \rho(A) = 2$

When $a = 1$, $A = \begin{vmatrix}0&0&0\\0&0&-7\\1&-2&2\end{vmatrix}$, $\therefore \rho(A) = 2$

When $a = 2$, $A = \begin{vmatrix}0&0&0\\0&0&-8\\1&-2&3\end{vmatrix}$ $\therefore \rho(A) = 2$

74 **(c)**

We have, $\begin{vmatrix}\sin x&\cos x&\cos x\\\cos x&\sin x&\cos x\\\cos x&\cos x&\sin x\end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix}\sin x + 2\cos x&\cos x&\cos x\\\sin x + 2\cos x&\sin x&\cos x\\\sin x + 2\cos x&\cos x&\sin x\end{vmatrix} = 0$$

$$\Rightarrow (2\cos x + \sin x)\begin{vmatrix}1&\cos x&\cos x\\1&\sin x&\cos x\\1&\cos x&\sin x\end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\Rightarrow (2\cos x + \sin x)\begin{vmatrix}1&\cos x&\cos x\\0&\sin x - \cos x&0\\0&0&\sin x - \cos x\end{vmatrix} = 0$$

$\Rightarrow (2\cos x + \sin x)(\sin x - \cos x)^2 = 0$

$\therefore \tan x = -2, 1$ But $\tan x \neq -2$, because it does not lie in the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$.

$\therefore \tan x = 1$

So, $x = \frac{\pi}{4}$

75 **(b)**

Given, angles of a triangle are A, B and C. We know that $A + B + C = \pi$, therefore

$A + B = \pi - C$

$\Rightarrow \cos(A + B) = \cos(\pi - C) = -\cos C$

$\Rightarrow \cos A\cos B - \sin A\sin B = -\cos C$

$\Rightarrow \cos A\cos B + \cos C = \sin A\sin B$ ...(i)

Let $\Delta = \begin{vmatrix}-1&\cos C&\cos B\\\cos C&-1&\cos A\\\cos B&\cos A&-1\end{vmatrix}$

$= -(1 - \cos^2 A) + \cos C(\cos C + \cos A\cos B) + \cos B(\cos B + \cos A\cos C)$

$= -\sin^2 A + \cos C(\sin A\sin B) + \cos B(\sin A\sin C)$ [from Eq.(i)]

$= -\sin^2 A + \sin A(\sin B\cos C + \cos B\sin C)$

$= -\sin^2 A + \sin A\sin(B + C)$

$= -\sin^2 A + \sin^2 A = 0$ $[\because \sin(B + C) = \sin(\pi - A) = \sin A]$

76 **(b)**

$\det(2A) = 2^4\det(A) = 16\det(A)$

77 **(a)**

Applying $R_1 \to R_1 + R_3 - 2R_2$, we get

$$\Delta = \begin{vmatrix}0&0&0&x + z - zy\\4&5&6&y\\5&6&7&z\\x&y&z&0\end{vmatrix}$$

$$= -(x+z-2y)\begin{vmatrix} 4 & 5 & 6 \\ 5 & 6 & 7 \\ x & y & z \end{vmatrix} \quad \text{[Expanding along } R_1\text{]}$$

$$= -(x+z-2y)\begin{vmatrix} 0 & -1 & 6 \\ 0 & -1 & 7 \\ x-2y+z & y-z & z \end{vmatrix}$$

$\begin{bmatrix} \text{Applying } C_1 \to C_1 + C_3 \\ -2C_2 \text{ and } C_2 \to C_2 - C_3 \end{bmatrix}$

$$= -(x+z-2y)^2 \begin{vmatrix} -1 & 6 \\ -1 & 7 \end{vmatrix} = (x-2y+z)^2$$

78 **(d)**

Clearly, $x = 0$ satifies the given equation

79 **(d)**

$$\Delta(-x) = \begin{vmatrix} f(-x)+f(x) & 0 & x^4 \\ 3 & f(-x)-f(x) & \cos x \\ x^4 & -2x & f(-x)f(x) \end{vmatrix}$$

$$\begin{vmatrix} f(x)+f(-x) & 0 & x^4 \\ 3 & f(x)-f(-x) & \cos x \\ x^4 & -2x & f(x)f(-x) \end{vmatrix} = -\Delta(x)$$

So, $\Delta(x)$ is an odd function.

$\Rightarrow$ $x^4\Delta(x)$ is an odd function

$$\Rightarrow \int_{-2}^{2} x^4\Delta(x)dx = 0$$

80 **(c)**

Applying $C_1 \to C_1 + C_2 + C_3$, we obtain

$$-x\begin{vmatrix} 1 & -6 & 3 \\ 1 & 3-x & 3 \\ 1 & 3 & -6-x \end{vmatrix} = 0$$

$$\Rightarrow -x\begin{vmatrix} 1 & -6 & 3 \\ 0 & 9-x & 0 \\ 0 & 9 & -9-x \end{vmatrix} = 0$$

$\begin{bmatrix} \text{Applying } R_2 \to R_2 - R_1, \\ R_3 \to R_3 - R_1 \end{bmatrix}$

$\Rightarrow -x(9-x)(-9-x) = 0 \Rightarrow x = 0, 9, -9$

Integer Answer Type

81 **(1)**

$$\mathrm{x + iy} = \begin{vmatrix} 3\mathrm{i} & -2\mathrm{i} & 1 \\ 2 & 2\mathrm{i} & -1 \\ 15 & 2 & \mathrm{i} \end{vmatrix}$$

$= 3\mathrm{i}(2\mathrm{i}^2 + 2) + 2\mathrm{i}(2\mathrm{i} + 15) + 1(4 - 30\mathrm{i})$

$= -6\mathrm{i} + 6\mathrm{i} - 4 + 30\mathrm{i} + 4 - 30\mathrm{i}$

$= 0$

$\Leftrightarrow \mathrm{x + iy} = 0 + \mathrm{i}0$

$\Leftrightarrow \mathrm{x = y} = 0$

$\Longrightarrow 1 + \mathrm{xy} = 1 + 0 = 1$

82 **(1)**

The given system of equations is consistent.

$$\Longrightarrow \begin{vmatrix} 1 & 1 & -3 \\ 1+\lambda & 2+\lambda & -8 \\ 1 & -(1+\lambda) & 2+\lambda \end{vmatrix} = 0$$

$\mathrm{C_2 \to C_2 - C_1 \text{ and } C_3 \to C_3 + 3C_1}$

$$\Longrightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1+\lambda & 1 & -5+3\lambda \\ 1 & -2-\lambda & 5+\lambda \end{vmatrix} = 0$$

$\Longrightarrow (5+\lambda) + (\lambda+2)(3\lambda-5) = 0$

$\Longrightarrow 5 + \lambda + 3\lambda^2 - 5\lambda + 6\lambda - 10 = 0$

$\Longrightarrow 3\lambda^2 + 2\lambda - 5 = 0$

$\Longrightarrow (3\lambda+5)(\lambda-1) = 0$

$\Longrightarrow \lambda = -\dfrac{5}{3}, \lambda = 1$

$\Longrightarrow \lambda = 1 \quad \text{.... [}\lambda \text{ is positive]}$

83 **(0)**

$$\Delta = \begin{vmatrix} x_1 & y_1 & 0 \\ x_2 & y_2 & 0 \\ x_3 & y_3 & 0 \end{vmatrix} \begin{vmatrix} y_1 & x_1 & 0 \\ y_2 & x_2 & 0 \\ y_3 & x_3 & 0 \end{vmatrix} = 0.0 = 0$$

84 **(2)**

System of equations

$\Rightarrow \alpha x + y + z = \alpha - 1$ (1)

$x + \alpha y + z = \alpha - 1$ (2)

$x + y + \alpha z = \alpha - 1$ (3)

Since system has no solution.

Therefore, (1) $\Delta = 0$ and (2) $\alpha - 1 \neq 0$

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0, \alpha \neq 1$$

$R_1 \to R_1 - R_3, R_2 \to R_2 \to R_3$

$$\begin{vmatrix} \alpha-1 & 0 & 1-\alpha \\ 0 & \alpha-1 & 1-\alpha \\ 1 & 1 & \alpha \end{vmatrix} = 0$$

$\Rightarrow (\alpha-1)[\alpha(\alpha-1) - (1-\alpha)] + (1-\alpha)[-(\alpha-1)] = 0$

$\Rightarrow (\alpha-1)[\alpha(\alpha-1) + (\alpha-1)] + (\alpha-1)^2 = 0$

$\Rightarrow (\alpha-1)^2[(\alpha+1)+1] = 0$

$\Rightarrow \alpha = 1, 1, -2 \Rightarrow \alpha = 1, -2$

Since system has no solution, $\alpha \neq 1$

$\therefore \alpha = -2$

85 **(4)**

$$\Delta = \begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = 0$$

Applying $R_3 \to R_3 - R_2$ and $R_2 \to R_2 - R_1$

$$\Delta = \begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ x+1 & x+1 & x+1 \\ x+2 & 2(x+2) & 6(x+2) \end{vmatrix} = 0$$

$$\therefore \ \Delta = (x+1)(x+2)\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 1 & 1 & 1 \\ 1 & 2 & 6 \end{vmatrix} = 0$$

$$\therefore \ \Delta = (x+1)(x+2)[(x+2).4 - (2x+3).5 + (3x+4).1] = 0$$

$\Delta = (x+1)(x+2)(-3x-3) = 0$

or $(x+1)^2(x+2) = 0$

$\therefore x = -1, -1, -2$